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## **Predicting the Duration of Leveraged Buyouts**

by

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*PREDICTING THE DURATION OF LEVERAGED BUYOUTS*

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### *Abstract*

We employ newly developed *split* hazard modeling to estimate the conditional probability that a firm will eventually return to public status following a leveraged buyout (LBO), and the conditional probability of reversion to public status in a given year for a firm that eventually may reverse. Our results, based on 343 LBO transactions, imply that not all LBO firms expect eventual reversion to public status. In addition, we find that those LBO decisions that are expected to enhance value the most are less likely to reverse eventually. We also find that eventual reversal probabilities and the timing of reversals for divisional LBOs are not significantly different from full-firm LBOs.

## PREDICTING THE DURATION OF LEVERAGED BUYOUTS

### I. INTRODUCTION

We employ modern developments in hazard rate modeling to predict the probability that a firm will eventually return to public ownership following a leveraged buyout (LBO), and to predict the conditional probability of reversal in a given year for a firm that eventually may reverse its LBO. Our findings may shed some light on the economic underpinnings of LBO reversals, and *ipso facto*, the economic motivation behind the controversial LBO decision itself. Of particular interest is our finding that not all LBOs are expected to reverse in the long run, a finding that is made possible by the choice of *split* hazard model estimation (introduced by Schmidt and Witte 1989). The conditional probability of reversal in subsequent periods increases with the passage of time since the buyout, and management buyouts (MBOs) reverse less quickly than other LBOs. An LBO transaction that is expected to enhance value the most, as measured by the stock price reaction to the announcement, is less likely to reverse eventually, and will reverse less quickly. We also find that the eventual reversal probabilities and the timing of reversals for divisional LBOs are not significantly different from full-firm LBOs.

Our study adds to the rapidly growing literature on LBOs in particular and financial restructuring in general. Prior studies have examined advantages and limitations of LBOs (Jensen 1986, 1989, Kaplan 1989b, Rappaport 1990), factors that induce firms to undertake

LBOs (Lehn and Poulsen 1989, Opler and Titman 1993), wealth effects of LBO decisions,<sup>1</sup> and post-buyout operating performance.<sup>2</sup> Most of the work on reverse LBOs has focused on post-IPO stock and operating performance (e.g., Degeorge and Zeckhauser 1993, Holthausen and Larcker 1993, and Mohan 1990). The studies that have examined the likelihood and timing of the reverse LBO decision include Degeorge and Zeckhauser (1993), Holthausen and Larcker (1993), and Kaplan (1991). Our study exploits a more complete information set on the timing of the reversal, as will be described below. Moreover, the adopted split hazard model allows us to assess in a nonparametric way the impact of the passage of time on a firm's conditional reversal probability, and it accounts for heterogeneity in reversal probabilities; i.e., it is not assumed that all firms have the same probability of eventual reversal.

In Section II, we frame our tests by appealing to various strands of economic theory pertinent to the LBO decision including (1) agency cost control (e.g., Jensen 1986, 1989 and Rappaport 1990); (2) efficient risk-bearing (e.g., Kaplan 1991); (3) efficient resource management (e.g., Fama and Jensen 1983, Hite and Vetsuypens 1989, and Kaplan 1991), and (4) information asymmetry (Kaplan 1991). The split hazard model is described and estimated in Section III, and we summarize and conclude in Section IV.

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<sup>1</sup>See Asquith and Wizman (1990), Cook, Easterwood and Martin (1992), DeAngelo, DeAngelo and Rice (1984), Harlow and Howe (1993), Hite and Vetsuypens (1989), Ippolito and James (1992), Kaplan (1989a), Kaplan and Stein (1990), Lowenstein (1985), Marais, Schipper and Smith (1989), Newbould, Chatfield and Anderson (1992), Shleifer and Summers (1988), and Smith (1990).

<sup>2</sup>See Kaplan (1989a), Lichtenberg and Siegel (1990), Muscarella and Vetsuypens (1990), and Smith (1990).

## II. THEORETICAL CONSIDERATIONS AND SAMPLE DESIGN

### A. The Duration of LBOs

Management ownership concentration reduces the stockholder-manager agency conflict (Jensen and Meckling 1976), hence Jensen (1986, 1989) argues that LBOs are superior to publicly held firms and should remain private for an unspecified, but significant amount of time. Rappaport (1990) argues, on the other hand, that leveraged going-private transactions offer reduced agency costs and improved operating efficiency through one-time changes. Once these changes are effected, the leverage poses a net disadvantage through reduced flexibility and vulnerability to financial distress. Add to this the desire for liquidity by LBO investors and owner-managers, and a strong motive emerges for eventual reversion to public ownership. Thus, a natural question is whether all LBO firms are expected to reverse eventually; i.e., what is the probability of ultimately returning to public ownership? A related but distinct question is, for those firms that eventually may reverse, what is the conditional probability of reversal in a given year? The split hazard model developed in the next section enables separate estimation of the probability of eventual reversal and the conditional probability of reversal in a given year.

Regardless of whether all LBOs are expected eventually to reverse, longevity of each may be due to firm-specific circumstances; i.e., the conditional probability of reversing in period  $t+1$  given private status through period  $t$  may vary according to observable characteristics. Kaplan (1991) notes that costs associated with risk-bearing plus the need for liquidity by LBO investors increase with LBO size, hence large LBOs should have shorter duration.

Divisional MBOs can lead to increases in efficiency in decision management and decision control (Fama and Jensen 1983). Hite and Vetsuypens (1989) explain how the change in ownership structure of a division can lead to improved resource allocation, and Kaplan (1991) argues that once the improvements have been realized, the division is likely to be taken public again. Thus, divisional buyouts should have shorter durations according to this argument. On the other hand, Kaplan (1991) argues that divisional managers are less likely to be able to exploit information asymmetries than managers of full firms that are taken private. Thus, divisional MBOs are less likely to go public again and may have longer durations.

The stock market reaction to LBO announcements can provide clues as to expected duration. Strong positive reactions should portend longer duration; i.e., firms that are expected to benefit most from private status should remain private longer. This translates into later reversals for LBOs with strong positive announcement reactions, and may also indicate that some LBOs will never reverse.

The foregoing discussion serves to identify several observable variables that may condition the probability of LBO reversal in a given period; i.e., the stock price reaction to the LBO announcement (measured by the cumulative abnormal return (CAR) over the announcement period), whether the LBO is for a division or a full firm (FULL is an indicator variable equal to 1 for full-firm buyouts and 0 otherwise); whether the buyout is management-led (MBO is an indicator variable equal to 1 for management-led buyouts and 0 otherwise), and the size of the LBO (SIZE is the transaction value in millions of dollars).



## B. The Sample

Our initial sample consisted of 483 LBO transactions completed in 1980-1992 with transaction size of \$100 million or more as reported by Securities Data Corporation. The following criteria resulted in 343 transactions:

1. LBO firms and parent firms of LBO divisions have stock returns in the Center for Research in Security Prices (CRSP) file during the calendar year prior to the LBO announcement.
2. Post-buyout status (public v. privately held) of the LBO is identifiable either in *Newspaper Abstracts* or *Ward's Business Directory of U.S. Private and Public Companies (1994)*.

Each LBO firm was classified as a completed or censored observation. Following Kaplan (1991), we define a completed observation as one in which the LBO firm or division returns to public status via an initial public offering (IPO) or upon acquisition by a publicly held firm. Censored observations were those which were still private and independent at the end of the observation period (October 31, 1993), were acquired by privately held firms, or had filed for bankruptcy.

The sample is described in Table 1. Of the full sample of 343 LBOs, 92 (26.8 percent) reversed by IPO after 42.5 months on average, and 31 (9 percent) were acquired by publicly held firms after about 46 months on average. Censored observations included 166 (48.4 percent) that were still private at the end of the observation period (October 31, 1993), 42 (12.2 percent) that were bankrupt, and 12 (3.5 percent) that had been acquired by privately held firms. Kaplan (1991) examines 183 LBOs, 14 percent of which reverse by IPO, 24 percent are acquired by publicly held firms, and 62 percent are still private at the end of the observation period. Thus,

our sample is similar to Kaplan's in terms of the proportion of censored observations (64.1 percent versus his 62 percent), but differs in that we have more IPOs and fewer public acquisitions.

[Insert Table 1]

The median duration for completed observations is 43 months, while the median for censored observations is 62 months. The average duration of our completed observations is roughly the same as that reported by Kidder and Peabody (1988), Muscarella and Vetsuypens (1990), and Kaplan (1991).

Of the 343 observations, 187 (54.5 percent) are full-firm LBOs ( $FULL=1$ ) and 156 (45.5 percent) are divisional ( $FULL=0$ ). MBOs represent 54.8 percent ( $MBO=1$ ) of the sample. The mean (median) SIZE of the LBO transactions is \$542 million (\$300 million), ranging from \$100 million to \$6.2 billion.

### **C. Valuation Effects of LBO Transactions**

In this section, we confirm that the LBOs in our sample are viewed on average by the capital markets as positive events. The announcement date ( $t=0$ ) is taken as the first date on which an LBO is mentioned in the press as recorded in *Newspaper Abstracts*. Abnormal returns are based on the market model with parameters estimated over the period  $t = -240$  to  $t = -39$ . Cumulative abnormal returns (CAR), cumulative average abnormal returns (CAAR), and test statistics (Z) are calculated identically to the method in Mikkelsen and Partch (1988). We omit observations for which 10 or more daily returns during the estimation period are missing.

In Table 2, we report CAAR, Z, the number of positive observations, and the significance level for the test for equal numbers of positive and negative CARs for various event windows. In Panel A of Table 2, we give the results for the 162 full-firm LBOs for which sufficient data were available, and in Panel B we report results for parent firm equity of 93 divisional LBOs.

[Insert Table 2]

For full-firm LBOs (Panel A), the CAAR is positive (15.6%) and highly significant ( $Z=55.53$ ) for the event period  $(-1, +1)$ , and this result holds over extended event periods as well. For example, for  $(-10, +10)$ , the CAAR is 20.2%, and highly significant ( $Z=26.61$ ). For each of the four event periods considered the percentage of positive CARs exceeds 90, and is highly significant in all cases based on the binomial probability distribution with equal probabilities of positive and negative CARs.

As expected, the results are less pronounced for parent firms (Panel B), but are nonetheless positive and significant at conventional levels. For the event period  $(-1, +1)$ , the CAAR is 2.1% ( $Z=4.599$ ), and for  $(-10, +10)$ , the CAAR is 2.6% ( $Z=3.138$ ). For  $(-1, +1)$ , 57% of CARs are positive (.0731 significance level), and for  $(-10, +10)$ , 61% are positive (.0110 level).

Our results are consistent with those in prior studies. For example, Lehn and Poulsen (1989) report a CAAR of 16.3% for the 3-day period around announcement (compare 15.6% in our study for the period  $(-1, +1)$ ), and Kaplan (1989) reports a CAAR of 26% (median value) for the 100-day period beginning 40 days before announcement.

### III. MODEL DEVELOPMENT AND ESTIMATION

#### A. The Split Hazard Function

We employ a hazard function  $h(t)$  to model the conditional probability that an LBO firm will reverse during period  $t$ , given that it has not done so as of period  $t - 1$ :

$$h(t) = \frac{f(t)}{1 - F(t)}, \quad (1)$$

where  $f(\cdot)$  and  $F(\cdot)$  are the respective probability density and cumulative distribution functions for  $t$ . A standard hazard model such as that employed by Kaplan (1991) relies on the implicit assumption that all LBO firms eventually reverse, although  $h(t)$  may vary across firms according to values of a vector of explanatory variables  $\mathbf{X}$ . A split hazard function allows for LBO firms to fall into either of two categories, (1) those that eventually will reverse, and (2) those that will never reverse. The split hazard model we develop below allows for heterogeneous reversal probabilities across firms, and our extension includes a nonparametric specification of the effect of time on firms' reversal probabilities.<sup>3</sup>

Define  $d_i$  as an indicator variable equal to 1 if the LBO firm has not reversed (censored observation), and equal to 0 if it has reversed. Denote by  $t_i$  the length of time that has lapsed from the LBO transaction to either the observed reversal date or the censoring date. Next define  $A_i$  as an unobservable indicator variable equal to 1 if firm  $i$  is an LBO that will eventually

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<sup>3</sup>Vanhuele, Dekimpe, Sharma and Morrison (1994) employ a standard hazard model modified to account for nonparametric time dependence and unobserved heterogeneous reversal probabilities. Our extension adds their modifications to the split hazard model of Schmidt and Witte (1989).

reverse, and equal to 0 if the LBO will never reverse. We express the likelihood of observing an LBO that has reversed in terms of the survival function  $S_i(\cdot)$ , the additive inverse of  $F_i(\cdot)$ , the cumulative distribution function of  $t_i$ :

$$P[d_i=0; t=t_i] = [S_i(t_i-1 | A_i=1) - S_i(t_i | A_i=1)] \times P(A_i=1). \quad (2)$$

In (2), the survival function  $S_i(\cdot)$  is conditioned on  $A_i = 1$ ; i.e.,  $S_i(\cdot)$  is the survival function for firms that eventually will reverse.

Similarly, the likelihood of observing an LBO firm with a censored duration ( $d_i = 1$ ) with time  $t_i$  having elapsed from the LBO transaction to the censoring date is given by

$$P[d_i=1; t=t_i] = P(A_i=0) + [P(A_i=1) \times S_i(t_i-1 | A_i=1)]. \quad (3)$$

Thus, for an arbitrary LBO firm  $i$ , the likelihood function is expressed as

$$L_i(t_i) = [\delta_i [S_i(t_i-1 | A_i=1) - S_i(t_i | A_i=1)]]^{(1-d_i)} \times [(1-\delta_i) + \delta_i S_i(t_i-1 | A_i=1)]^{d_i}, \quad (4)$$

where  $\delta_i \equiv P(A_i = 1)$ , hence  $1 - \delta_i = P(A_i = 0)$ , the probability that the LBO firm will never reverse.

Our next step is to recognize that the survival probability  $S_i(t_i)$  at any given time may depend on firm-specific characteristics  $X_i(t)$  as well as the duration of the LBO up to period  $t_i$ . Following the usual procedure (Lancaster 1990; Tuma and Hannan 1984), we first develop an expression for the reversal or hazard rate function  $\lambda_i(t)$ , then use the unique relationship between

$\lambda_i(t)$  and  $S_i(t)$  to derive the likelihood function.<sup>4</sup> Following Vanhuele, *et al.* (1994), we define  $D_i(t)$  as a vector of time-varying indicator variables. For example,  $D_i(3)$  is 1 in the third year an LBO is private, and 0 during all other years. This nonparametric specification may be interpreted as a piecewise approximation to the underlying continuous baseline hazard, and allows for monotonic or nonmonotonic time dependence in the reversal rate, which is modeled as:

$$\lambda_i(t) = \lambda_0 e^{\beta X_i(t)} e^{C D_i(t)}. \quad (5)$$

In (5),  $\lambda_0$  is the baseline hazard; i.e., the reversal rate in the first post-buyout period for the base group of firms for which all elements of  $X_i(t)$  are 0. The explanatory variables  $X_i(t)$  may be fixed or time-varying. The coefficient vector  $\beta$  reflects the effects of the explanatory variables, and the vector  $C$  measures the impact of the passage of time on the reversal probability starting with the second period. The hazard function in (5) is subsequently transformed to the survivor function as described in footnote 4. When substituted into equation (4), the conditional likelihood contribution  $L_i(t_i | \lambda_0)$  is obtained. We term  $L_i(t_i | \lambda_0)$  "conditional" because it depends on a specific value of  $\lambda_0$ .

The specification thus far still may not account fully for the heterogeneity in the reversal rate. Kaplan (1991, p. 298) notes that explicit control for unobserved heterogeneity is necessary to avoid incorrect inferences as to the true time-dependency of the reversal probability, but his method does not take such heterogeneity into account. For example, suppose every LBO firm

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<sup>4</sup> $S_i(t)$  may be written as  $\exp(-\theta_i(t))$ , where  $\theta_i(t)$  is the integral of  $\lambda_i(t)$  from 0 to  $t$ . See Ross (1980).

has a *constant* reversal probability over time, but some firms' probabilities are high and some are low. In such a case the LBOs with high reversal probabilities will likely reverse after a few periods, leaving a higher proportion of LBO firms with low reversal probabilities. Thus, fewer reversals will be observed as time passes giving the incorrect impression that reversal probabilities are *decreasing* over time.<sup>5</sup> Schmittlein and Morrison (1983) approach the problem by allowing the baseline hazard  $\lambda_0$  to vary across firms according to a certain distribution. Their approach is to weight the conditional likelihood by the relative occurrence of its  $\lambda_0$  value by means of a mixing distribution  $g(\lambda_0)$ . Thus the unconditional likelihood becomes

$$L_i(t_i) = \int_0^{\infty} L_i(t_i | \lambda_0) g(\lambda_0) d\lambda_0. \quad (6)$$

The resulting expression for  $L_i(t_i)$  in (6) will account for firm-specific characteristics  $\mathbf{X}_i(t)$ , the passage of time  $D_i(t)$ , and unobserved heterogeneity in the baseline reversal rate through  $g(\lambda_0)$ . Using the gamma distribution as the mixing distribution, it can be shown that the log-likelihood function for  $N$  firms is

$$\begin{aligned} LL = \sum_{i=1}^N \ln \left\{ \frac{(\delta_i^{1-d_i} - \delta_i)(1+d_i)a^r}{[(1-d_i)B_i(t_i-1) + a]^r} - \frac{(\delta_i^{1-d_i} - \delta_i)a^r}{[(1-d_i)B_i(t_i) + a]^r} \right. \\ \left. + \frac{\delta_i(1+d_i)a^r}{[B_i(t_i-1) + a]^r} - \frac{\delta_i a^r}{B_i(t_i-1) + (1-d_i)e^{\beta \mathbf{X}_i(t_i) + cD_i(t_i)} + a]^r} \right\}, \quad (7) \end{aligned}$$

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<sup>5</sup>Ignoring unobserved heterogeneity introduces bias in the estimation of  $C$  in equation (5), and also renders estimates of  $\beta$  biased and inconsistent. See Heckman and Singer (1984a, 1984b), and Lancaster (1990).

where  $\delta_i = P(A_i = 1)$ ,  $B_i(t_i) = \sum_{j=1}^{t_i} \exp[\beta X_i(j) + cD_i(j)]$ , and  $a$  and  $r$  are the location and shape parameters, respectively, of the gamma distribution. The mean of the gamma distribution is given by  $E(\lambda_0) = r/a$ . The coefficient estimates of the  $\beta$  and  $C$  vectors should then be interpreted relative to  $r/a$ .

If we assume that every LBO firm has the same probability of eventual reversal,  $P(A_i = 1)$ , then  $\delta_i = \delta$  in (7), and the operation is simplified. However, it seems more plausible that the eventual reversal probability will vary across firms based on a set of firm characteristics  $X_i$ , such that

$$P(A_i=1) = \delta_i = \delta(X_i). \quad (8)$$

We use a logit specification to model  $\delta_i$  as follows:

$$\delta_i = \delta(X_i) = \frac{1}{1 + e^{-\alpha X_i}}. \quad (9)$$

Thus the full split hazard model allows us to estimate the effect of explanatory variables  $X_i$  on the probability of eventual reversal (through  $\alpha$  in (9)), or the effect of  $X_i$  on the timing (through  $\beta$  in (7)).

In the case where  $P(A_i = 1) = \delta$ , constant across firms,  $\delta$  represents the proportion of LBO firms that are temporary; i.e., that eventually will reverse. In the case where  $\delta = 1$ , we see that the split hazard model collapses to the standard hazard model where all firms are expected to reverse eventually. Even though  $\delta$  may be the same for all firms, duration is then still allowed to vary across firms due to differential effects of  $\beta X_i$ .



## B. Estimation Results

In Table 3, we present results of maximizing the log-likelihood function (7) for the full sample of 343 observations. Because 156 of these are divisional, the model is estimated first without CAR values.

[Insert Table 3]

The parameters estimated in Table 3 include  $r/a$ , the coefficient for FULL (= 1 if full firm, 0 if divisional), MBO (= 1 if management participation, 0 otherwise), the coefficient for SIZE (transaction dollar value/100 million), and the vector  $C$ . The latter contains  $(C_2, \dots, C_7)$ , where  $C_2$  corresponds to year 2 (13-24 months),  $C_3$  corresponds to year 3 (25-36 months), and so on. In Table 3, we present results in column (1) for the case of homogeneous reversal probability ( $\delta$ ), and in column (2) we report parameter estimates for heterogeneous reversal probability ( $\delta(X_i)$ ).

For the homogeneous eventual reversal probability case (column (1)), the estimated mean reversal probability for the base year is 6.1%; i.e., the reversal probability in the first year for divisional buyouts (FULL = 0) without management participation (MBO = 0). The C-coefficients are interpreted relative to the first post-buyout year, thus each C-coefficient reflects a *proportional* shift in the conditional reversal rate relative to the first post-buyout year. For example,  $C_3$  is estimated at .589. For a non-MBO observation (MBO = 0), for a division (FULL = 0), the probability of reversal in year 3 is  $.061 \times \exp(.589) = .11$ . Then,  $C_4 = 1.176$ , hence the reversal probability for such a firm in year 4 is  $.061 \times \exp(1.176) = .20$ , indicating an economically meaningful shift in the fourth year. We reject the joint null

hypothesis,  $C_2 = C_3 = \dots = C_7 = 0$ , at the .01 level based on the approximate  $\chi^2$  criterion, thus there is evidence of significant time dependence.<sup>6</sup> All individual C-coefficient estimates except  $C_2$  are significantly different from zero at the .05 level. Thus, as the number of years since the buyout increases, the conditional reversal probability remains positive and exceeds that for the first year. We also find that the estimates of  $(C_4, \dots, C_7)$  exceed those of  $(C_2, C_3)$  significantly, thus the conditional reversal probability increases from years 1-3 to years 4-13. After the fourth year following the buyout, the conditional reversal probability levels off (we cannot reject  $C_4 = C_5 = C_6 = C_7$ ), but is still positive and significantly greater than in the first 3 years. In Figure 1, we depict the conditional probability of reversal for years 1 through 7 for the case where  $MBO = FULL = 0$ . The heavy line is a smoothed approximation of the estimated (discrete) function.<sup>7</sup> This illustrates the sharp increase in reversal probability in year 4.

[Insert Figure 1]

Given the homogeneity assumption, we conclude that some buyouts are expected to reverse later than others; i.e., MBOs are expected to reverse *less* quickly (coefficient estimate = -.334, significant at the .10 level based on the likelihood ratio test). We show in Figure 1 the conditional reversal probability for a divisional MBO ( $MBO = 1, FULL = 0$ ). The probability function (light line) lies everywhere below that for  $MBO = 0$ , and does not rise as sharply by the end of year 4. This illustrates that MBOs tend to reverse less quickly than non-

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<sup>6</sup>See Judge *et al.* (1988), p. 105.

<sup>7</sup>The discrete function is smoothed by cubic spline interpolation.

MBOs, and this is consistent with the argument by Kaplan (1991) based on asymmetric information.

The model reveals a negative but insignificant estimate for the effect of FULL on reversal probability. Thus, we cannot conclude that full-firm LBOs tend to reverse more quickly than divisional LBOs (coefficient estimate =  $-.187$ , not significant at the  $.15$  level). The effect of transaction SIZE is positive but insignificant at the  $.15$  level. Maximization of (7) under the assumption of homogeneous  $\delta$  allows us to estimate that parameter. The point estimate is  $.882$  and we cannot reject  $\delta = 1$  at the  $.15$  level.

Thus far, based on the results in Table 3, column (1), we may conclude that significant positive time-dependency exists, and that MBOs are expected to reverse earlier than other LBOs. In column (2) of Table 3, we allow for heterogeneity in  $\delta$ ; i.e.,  $\delta_i = \delta(X_i)$  as in (9). The explanatory variables SIZE, FULL and MBO now enter the model through the logit specification in (9), and given the functional form the signs of coefficients have opposite interpretations. Again we note significant positive time-dependency; i.e., the elements of  $C$  are nonzero jointly and individually except for  $C_2$ . Thus, conditional reversal probability increases with passage of time since the LBO, and the patterns and magnitudes of the C-coefficient estimates are quite similar to those in column (1). As in the case of homogeneous  $\delta$ , we continue to find insignificant effects of FULL and SIZE, but the probability of eventual reversal is significantly smaller for MBOs (coefficient estimate =  $15.586$ , significant at the  $.05$  level).

Thus far we have seen that management participation has a significant impact on the *timing* of the reversal and on the probability of *eventual* reversal, thus it appears that not all LBOs have the same eventual reversal probability ( $\delta$ ), even though the average  $\delta$  may be high.

In Table 4, we present model estimates for 160 full-firm LBOs, thus we are able to examine the effects on duration and eventual reversal probability of the market's reaction to LBO announcements. The market's reaction is measured as the CAR for the period  $t = -5$  to  $t = +5$ . As before we find evidence of significant time dependency; i.e., coefficient estimates  $C_2$ ,  $C_3$ ,  $C_4$ , and  $C_5$  are jointly significant at the 5 percent level. Note that the C-coefficients are defined differently than in Table 3. Due to the composition of the (smaller) sample of full-firm LBOs, we define  $C_2, \dots, C_5$  commencing after the third year since the LBO. Assuming homogeneous probability ( $\delta$ ) of eventual reversal (column (1), Table 4), we find that MBO exerts a significant effect on duration; MBOs are again expected to reverse *less* quickly. But when we allow for heterogeneous eventual reversal probability, we find that MBO has an insignificant effect on this probability; i.e., MBOs do not exhibit lower eventual reversal probabilities, but they tend to reverse less quickly.

[Insert Table 4]

The effect of the LBO announcement effect (CAR) on duration is significant at the 5 percent level; i.e., large positive reactions portend reversal less quickly. Thus, LBOs that promise to be the most valuable reorganizations (high CARs) tend to remain private longer. In column (2) of Table 4, we allow for heterogeneous eventual reversal probability. The probability of eventual reversal ( $\delta$ ) is significantly smaller for LBOs that produced high CAR values at announcement. Thus, allowing for heterogeneous eventual reversal probability, we may conclude that LBO announcements that are greeted most enthusiastically by the stock market

are also those that tend to have lower eventual reversal probabilities, and those that tend to reverse less quickly.

#### IV. SUMMARY AND CONCLUSIONS

We employ a large sample (343 observations) of LBOs that took place during the period 1980-1992 to investigate the duration of the LBOs and their probability of eventual reversal. Hazard rate methods are used because they allow for censored data; i.e., some LBO firms go bankrupt, are taken over by privately held firms, or continue to be private at the end of the observation period. We employ a split hazard model that allows for the probability of eventual reversal to vary across firms; earlier work by Kaplan (1991) uses standard hazard rate methods which assume implicitly that all LBOs eventually reverse, hence the (homogeneous) probability of eventual reversal is 1. In addition, we employ the stock price reaction to the LBO announcements as an explanatory variable.

Our results reveal pronounced time dependence in the reversal rates; i.e., the conditional probability of reversal in period  $t + 1$  depends positively on the length of time  $t$  that the firm has been private. We find that MBOs tend to reverse less quickly in the short run and that they have lower eventual reversal probabilities, thus not all firms are expected to reverse eventually. We also find that those firms whose going-private announcements produced large positive stock price reactions tend to reverse less quickly, and their eventual reversal probabilities are significantly lower than other firms. These findings are evidently new and may help us to understand more fully the economic motives and consequences of LBO transactions.

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Table 1

**Description of Sample of 343 LBOs, 1980-1992:  
Duration of Censored and Completed Observations, SIZE,  
Full/Divisional Buyouts and Management Participation (MBO)**

	N	Mean (Months)	Median (Months)	Min	Max
<b>A. <u>Duration</u></b>					
Completed					
IPO	92	42.50	42.50	4	109
Acquired	31	46.03	46.00	4	152
Censored					
Private	166	66.70	64.00	12	157
Bankrupt	42	46.93	46.50	14	89
Acquired	12	51.00	45.50	15	118
		Mean	Median	Min	Max
<b>B. <u>Other Characteristics</u></b>					
SIZE (\$ million)		542	300	100	6200
FULL Firm		187/343 = 54.5%			
MBO		188/343 = 54.8%			

Table 2

**Stock Price Reactions to LBO Announcements, 1980-1992:  
Cumulative Average Abnormal Returns (CAAR), Test Statistics (Z),  
Percentage of Positive Cumulative Abnormal Returns (CAR), and  
Significance Level for Rejection of Hypothesis of  
Equal Positive and Negative CARs**

Event Window	CAAR (%)	Z	% Pos	P-Value
<b>A. <u>Full Firm LBOs</u></b>				
[-1, +1]	15.6	55.530	90	.0000
[-3, +3]	18.1	41.859	94	.0000
[-5, +5]	19.2	35.337	94	.0000
[-10, +10]	20.2	26.610	92	.0000
<b>B. <u>Parent Firms of Divisional LBOs</u></b>				
[-1, +1]	2.1	4.599	57	.0731
[-3, +3]	3.3	5.177	63	.0034
[-5, +5]	2.5	3.382	56	.1066
[-10, +10]	2.6	3.138	61	.0110

Table 3

**Parameter Estimates of Split Hazard Model  
of Duration of 343 LBOs, 1980-1992**

Explanatory variables are FULL (= 1 if full-firm, 0 if division); MBO (= 1 if MBO, 0 otherwise); SIZE (in dollars/1000); and time ( $C_2, \dots, C_7$ ).

Parameter	(1)	(2)
	Homogeneous Reversal Probability	Heterogeneous Reversal Probability
r/a	0.061	0.046
$C_2$ (13-24 mos.)	0.253	0.250
$C_3$ (25-36 mos.)	0.589**	0.578**
$C_4$ (37-48 mos.)	1.176***	1.161***
$C_5$ (49-60 mos.)	1.325***	1.308***
$C_6$ (61-72 mos.)	1.249***	1.213***
$C_7$ (73+ mos.)	1.354***	1.247***
FULL	-0.187	—
MBO	-0.334**	—
SIZE	0.023	—
Homogeneous $\delta$	0.882	—
Heterogeneous $\delta(X_i)$		
Intercept	—	-18.277***
FULL	—	1.437
MBO	—	15.586***
SIZE	—	0.452
N	343	343
LL	-411.934	-411.418

\*\*Significant at the 10% level, based on likelihood ratio tests.

\*\*\*Significant at the 5% level, based on likelihood ratio tests.

Table 4

Parameter Estimates of Split Hazard Model of Duration  
of 160 Full-firm LBOs. Explanatory variables are  
MBO (= 1 if MBO, 0 otherwise); CAR (announcement period  
abnormal return), and time ( $C_2, C_3, \dots, C_5$ )

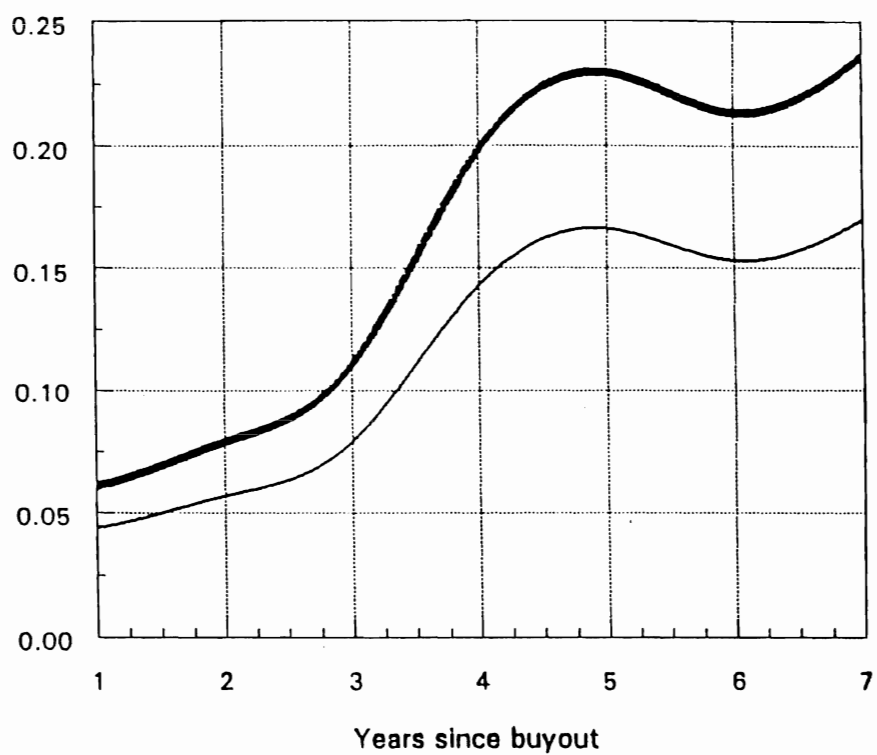
Parameter	(1) Homogeneous Reversal Probability	(2) Heterogeneous Reversal Probability
r/a	0.041	0.037
$C_2$ (37-48 mos.)	1.530***	1.578***
$C_3$ (49-60 mos.)	1.866***	1.973***
$C_4$ (61-72 mos.)	1.516***	1.681***
$C_5$ (73+ mos.)	1.628***	1.978***
SIZE	0.103	---
MBO	-0.419**	---
CAR	-1.738**	---
Homogeneous $\delta$	0.876	---
Heterogeneous $\delta(X_i)$ :		
Intercept	---	-2.686***
SIZE	---	-0.093
MBO	---	0.826
CAR	---	5.612***
N	160	160
LL	-186.985	-187.108

\*\*Significant at the 10% level, based on likelihood ratio tests.

\*\*\*Significant at the 5% level, based on likelihood ratio tests.

Figure 1

Estimated conditional probability of reversal of LBOs in  $j$ th year following LBO transaction,  $j = 1, 2, \dots, 7$ . The heavy line represents non-MBO division buyouts, while the light line is for MBOs.



# DERIVATION OF THE SPLIT HAZARD MODEL WITH GAMMA MIXING DISTRIBUTION

From equation (4) in the text, the conditional likelihood function for the  $i$ -th LBO is equal to

$$L_i(t_i|\lambda_0) = [\delta_i \{S_i(t_{i-1}|\lambda_0; A_i=1) - S_i(t_i|\lambda_0; A_i=1)\}]^{1-d_i} \times [1 - \delta_i + \delta_i S_i(t_{i-1}|\lambda_0; A_i=1)]^{d_i} . \quad (A.1)$$

Assuming that the explanatory variables remain constant within each period but are allowed to vary across periods, it can be shown that (Ross 1980):

$$S_i(t_i|\lambda_0) = e^{-\int_0^{t_i} \lambda_i(u) du} = e^{-\lambda_0 B_i(t_i)} , \quad (A.2)$$

where

$$B_i(t_i) = \sum_{j=1}^{t_i} e^{[\beta X_i(j) + c D_i(j)]} . \quad (A.3)$$

Hence,

$$L_i(t_i|\lambda_0) = [\delta_i (e^{-\lambda_0 B_i(t_{i-1})} - e^{-\lambda_0 B_i(t_i)})]^{1-d_i} \times [1 - \delta_i + \delta_i e^{-\lambda_0 B_i(t_{i-1})}]^{d_i} . \quad (A.4)$$

Since

$$B_i(t_i) = B_i(t_{i-1}) + e^{\beta X_i(t_i) + c D_i(t_i)} , \quad (A.5)$$

we can write

$$(e^{-\lambda_0 B_i(t_{i-1})} - e^{-\lambda_0 B_i(t_i)}) = e^{-\lambda_0 B_i(t_{i-1})} [1 - e^{-\lambda_0 e^{\beta X_i(t_i) + c D_i(t_i)}}] , \quad (A.6)$$

and equation (A.4) becomes

$$L_i(t_i|\lambda_0) = [\delta_i e^{-\lambda_0 B_i(t_i-1)} (1 - e^{-\lambda_0 e^{\beta X_i(t_i) + c D_i(t_i)}})]^{1-d_i} \times [1 - \delta_i + \delta_i e^{-\lambda_0 B_i(t_i-1)}]^{d_i}. \quad (A.7)$$

Noting that  $d_i$  can only take the value of zero or one, the last term of equation (A.7) can be written as

$$[1 - \delta_i + \delta_i e^{-\lambda_0 B_i(t_i-1)}]^{d_i} = 1 - \delta_i^{d_i} + [\delta_i e^{-\lambda_0 B_i(t_i-1)}]^{d_i}. \quad (A.8)$$

The unconditional likelihood for the  $i$ -th LBO, which is obtained by weighing the corresponding conditional likelihood by the relative occurrence of its  $\lambda_0$ -value, is equal to

$$\begin{aligned} L_i(t_i) &= \int_0^\infty L_i(t_i|\lambda_0) g(\lambda_0) d\lambda_0 \\ &= \int_0^\infty \{ [\delta_i e^{-\lambda_0 B_i(t_i-1)} (1 - e^{-\lambda_0 e^{\beta X_i(t_i) + c D_i(t_i)}})]^{1-d_i} \\ &\quad \times [1 - \delta_i^{d_i} + \delta_i^{d_i} e^{-\lambda_0 d_i B_i(t_i-1)}] \} g(\lambda_0) d\lambda_0. \end{aligned} \quad (A.9)$$

Multiplying out expression (A.9):

$$\begin{aligned}
L_i(t_i) &= \int_0^\infty [\delta_i e^{-\lambda_0 B_i(t_i-1)} (1 - e^{-\lambda_0 e^{\beta_{X_i}(t_i)} + c_{D_i}(t_i)})]^{1-d_i} g(\lambda_0) d\lambda_0 \\
&\quad - \int_0^\infty [\delta_i e^{-\lambda_0 B_i(t_i-1)} (1 - e^{-\lambda_0 e^{\beta_{X_i}(t_i)} + c_{D_i}(t_i)})]^{1-d_i} \delta_i^{d_i} g(\lambda_0) d\lambda_0 \\
&\quad + \int_0^\infty [\delta_i e^{-\lambda_0 B_i(t_i-1)} (1 - e^{-\lambda_0 e^{\beta_{X_i}(t_i)} + c_{D_i}(t_i)})]^{1-d_i} \delta_i^{d_i} e^{-\lambda_0 d_i B_i(t_i-1)} g(\lambda_0) d\lambda_0 .
\end{aligned} \tag{A.10}$$

Recalling that  $d_i$  can only take the value of zero or one, the first term of equation (A.10) can be written as:

$$\begin{aligned}
&\int_0^\infty \{ [\delta_i e^{-\lambda_0 B_i(t_i-1)}]^{1-d_i} \\
&\quad \times [(1+d_i) - (e^{-\lambda_0 e^{\beta_{X_i}(t_i)} + c_{D_i}(t_i)})^{1-d_i}] \} g(\lambda_0) d\lambda_0 ,
\end{aligned} \tag{A.11}$$

which is equal to

$$\begin{aligned}
&\int_0^\infty \delta_i^{1-d_i} e^{-\lambda_0 (1-d_i) B_i(t_i-1)} (1+d_i) g(\lambda_0) d\lambda_0 \\
&\quad - \int_0^\infty \delta_i^{1-d_i} e^{-\lambda_0 (1-d_i) B_i(t_i-1)} [e^{-\lambda_0 e^{\beta_{X_i}(t_i)} + c_{D_i}(t_i)}]^{1-d_i} g(\lambda_0) d\lambda_0 .
\end{aligned} \tag{A.12}$$

Substituting the gamma mixing distribution for  $g(\lambda_0)$  such that

$$g(\lambda_0; r, a) = \left[ \frac{a}{\Gamma(r)} \right] [a\lambda_0]^{r-1} [e^{-a\lambda_0}] \quad \lambda_0, r, a > 0 , \tag{A.13}$$



the first term of equation (A.12) becomes

$$\begin{aligned}
& \int_0^{\infty} \delta_i^{1-d_i} (1+d_i) e^{-\lambda_0 (1-d_i) B_i(t_i-1)} \frac{a^r}{\Gamma(r)} e^{-a \lambda_0} \lambda_0^{r-1} d\lambda_0 \\
&= \frac{\delta_i^{1-d_i} (1+d_i) a^r}{\Gamma(r)} \int_0^{\infty} e^{-\lambda_0 [(1-d_i) B_i(t_i-1) + a]} \lambda_0^{r-1} d\lambda_0 \\
&= \frac{\delta_i^{1-d_i} (1+d_i) a^r}{\Gamma(r)} \int_0^{\infty} e^{-\lambda_0 [(1-d_i) B_i(t_i-1) + a]} \frac{[(1-d_i) B_i(t_i-1) + a]^r}{[(1-d_i) B_i(t_i-1) + a]^r} \lambda_0^{r-1} d\lambda_0 \\
&= \frac{\delta_i^{1-d_i} (1+d_i) a^r}{\Gamma(r)} \times \frac{\Gamma(r)}{[(1-d_i) B_i(t_i-1) + a]^r} , \tag{A.14}
\end{aligned}$$

where  $\Gamma(r)$  is the gamma function. The last expression was obtained by recognizing that

$$\begin{aligned}
& \int_0^{\infty} e^{-\lambda_0 [(1-d_i) B_i(t_i-1) + a]} [(1-d_i) B_i(t_i-1) + a]^r \lambda_0^{r-1} d\lambda_0 \\
&= \int_0^{\infty} e^{-\lambda_0 [(1-d_i) B_i(t_i-1) + a]} \\
&\quad \times [\lambda_0 ((1-d_i) B_i(t_i-1) + a)]^{r-1} d[\lambda_0 ((1-d_i) B_i(t_i-1) + a)] \\
&= \Gamma(r) . \tag{A.15}
\end{aligned}$$

Using equation (A.5), the second term of equation (A.12) can be written as

$$\int_0^{\infty} \delta_i^{1-d_i} e^{-\lambda_0 (1-d_i) B_i(t_i)} g(\lambda_0) d(\lambda_0) , \quad (\text{A.16})$$

which, using the same algebraic manipulations as above, is equal to

$$\frac{\delta_i^{1-d_i} a^r}{[(1-d_i) B_i(t_i) + a]^r} . \quad (\text{A.17})$$

Hence, the first term in equation (A.10) is equal to

$$\frac{\delta_i^{1-d_i} (1+d_i) a^r}{[(1-d_i) B_i(t_{i-1}) + a]^r} - \frac{\delta_i^{1-d_i} a^r}{[(1-d_i) B_i(t_i) + a]^r} . \quad (\text{A.18})$$

Using a similar logic, the second term in equation (A.10) can be shown to equal

$$\frac{\delta_i (1+d_i) a^r}{[(1-d_i) B_i(t_{i-1}) + a]^r} - \frac{\delta_i a^r}{[(1-d_i) B_i(t_i) + a]^r} . \quad (\text{A.19})$$

Finally, the third term in equation (A.10) can be written as

$$\int_0^{\infty} \delta_i e^{-\lambda_0 B_i(t_{i-1})} [1 - e^{-\lambda_0 e^{\beta X_i(t_i) + c D_i(t_i)}}]^{1-d_i} g(\lambda_0) d\lambda_0 . \quad (\text{A.20})$$

Noting that

$$[1 - e^{-\lambda_0 e^{\beta X_i(t_i) + c D_i(t_i)}}]^{1-d_i} = (1+d_i) - [e^{-\lambda_0 e^{\beta X_i(t_i) + c D_i(t_i)}}]^{1-d_i} , \quad (\text{A.21})$$

the expression for (A.20) becomes

$$\begin{aligned} & \delta_i \int_0^{\infty} e^{-\lambda_0 B_i(t_{i-1})} (1+d_i) g(\lambda_0) d\lambda_0 \\ & - \delta_i \int_0^{\infty} e^{-\lambda_0 B_i(t_{i-1})} [e^{-\lambda_0 e^{\beta X_i(t_i) + c D_i(t_i)}}]^{1-d_i} g(\lambda_0) d\lambda_0 . \end{aligned} \quad (\text{A.22})$$

Substituting the gamma mixing distribution for  $g(\lambda_0)$ , the first part of equation (A.22) becomes

$$\begin{aligned}
& \delta_i \int_0^{\infty} e^{-\lambda_0 B_i(t_{i-1})} (1+d_i) \frac{a^r}{\Gamma(r)} e^{-a\lambda_0} \lambda_0^{r-1} d\lambda_0 \\
&= \delta_i (1+d_i) \frac{a^r}{\Gamma(r)} \int_0^{\infty} \frac{e^{-\lambda_0 [B_i(t_{i-1})+a]} [B_i(t_{i-1})+a]^r \lambda_0^{r-1}}{[B_i(t_{i-1})+a]^r} d\lambda_0 \quad (\text{A.23}) \\
&= \delta_i (1+d_i) \frac{a^r \Gamma(r)}{\Gamma(r) [B_i(t_{i-1})+a]^r} \\
&= \delta_i (1+d_i) \left[ \frac{a}{B_i(t_{i-1})+a} \right]^r,
\end{aligned}$$

and the second part of equation (A.22) becomes

$$\begin{aligned}
& \delta_i \int_0^{\infty} e^{-\lambda_0 B_i(t_{i-1})} [e^{-\lambda_0 e^{\beta X_i(t_i) + c D_i(t_i)}}]^{1-d_i} g(\lambda_0) d\lambda_0 \\
&= \delta_i \int_0^{\infty} e^{-\lambda_0 [B_i(t_{i-1}) + (1-d_i) e^{\beta X_i(t_i) + c D_i(t_i)} + a]} \frac{a^r}{\Gamma(r)} \lambda_0^{r-1} d\lambda_0 \quad (\text{A.24}) \\
&= \delta_i \frac{a^r \Gamma(r)}{\Gamma(r) [B_i(t_{i-1}) + (1-d_i) e^{\beta X_i(t_i) + c D_i(t_i)} + a]^r}.
\end{aligned}$$

Combining equations (A.23) and (A.24), the new expression for (A.20) becomes:

$$\frac{\delta_i (1+d_i) a^r}{[B_i(t_{i-1})+a]^r} - \frac{\delta_i a^r}{[B_i(t_{i-1}) + (1-d_i) e^{\beta X_i(t_i) + c D_i(t_i)} + a]^r}. \quad (\text{A.25})$$

Combining the three terms of the unconditional likelihood function (equations (A.18), (A.19) and (A.25)), the log-likelihood function for  $N$  LBOs is equal to

